OSCILLATIONS and WAVES

1. Circular motion

Rotation Units
1 rev = 2\pi rad = 360 deg

Definitions
Period (T): Time of one complete rotation. Unit: sec.
Frequency (f, \nu): Number of revolutions in one second. Unit: rev/s = Hz
Therefore: \[ f = \frac{1}{T} \]
\[ f = \frac{N}{t} \quad \text{and} \quad T = \frac{t}{N} \quad (N: \text{number of revs}) \]

Angular velocity (\omega): Angle rotated in one sec.
\[ \omega = \frac{\theta}{t} \Rightarrow \text{Unit: rad/s} \]

[Angular velocity is sometimes called angular frequency (or cyclic frequency) because:]

Note: Frequency and \omega actually measure the same quantity (rotation rate) only with different units.
[f is how many rotations in one second, \omega is how many radians in one second]
\[ f = \frac{\text{rev}}{s} \quad ; \quad \omega = \frac{\text{rad}}{s} \]

Since 1 rev = 2\pi rad
\[ f = 1 \text{ rev/s} \Rightarrow \omega = 2\pi \text{ rad/s} \]
\[ f = 2 \text{ rev/s} \Rightarrow \omega = 4\pi \text{ rad/s} \]
\[ f = 3 \text{ rev/s} \Rightarrow \omega = 6\pi \text{ rad/s} \]
\[ \omega = 2\pi f = \frac{2\pi}{T} \]

2. Harmonic oscillations

Definitions
Equilibrium point: Where F=0
Displacement (y): Distance from equilibrium point.
Changes with time: \[ y = y(t) \]
Amplitude (A): Maximum displacement from equilibrium. \[ A = \text{constant if no friction} \]
Period (T): Time to complete one cycle
Frequency (f): Number of oscillations in one sec.

One oscillation: \[ \begin{cases} +A \\ -A \end{cases} \]

Equation of displacement:
If motion starts from equilibrium (at t=0 y=0)
\[ y(t) = A \sin(\omega t) \]
A: Amplitude
\[ \omega: \text{Angular frequency (}\omega = 2\pi f\text{)} \]
(\omega t): phase (angle)

Relation with circular motion:
\[ y = A \sin(\omega t) \]

Ex: Oscillation of a mass-spring system is given by
\[ y(t) = 10 \sin(8\pi t) \text{ in centimeters.} \]
(a) Find amplitude, frequency, and period.
(b) Find distance from eq. at t = 0.5 s
(c) Find phase of oscillation at t=0.5 s

Ex: Mass-spring system makes 5 complete oscillations
in 10 s between two points 14 cm apart starting from equilibrium. Write equation y(t)=?

Ex: m=4kg mass oscillates at the end of k= 100 N/m spring. Write equation y(t)=? if amplitude is 18 cm.

Phase difference (\Delta\phi):
Two systems oscillating same frequency one started \Delta t earlier (or earlier).

\[ y_1(t) = A \sin(\omega t) \]
\[ y_2(t) = A \sin(\omega t + \Delta\phi) \]

Ex: Two mass-spring systems oscillating with same period T=4 s and A=10 cm. One starts at t=0 the other \Delta t=1 s later from equilibrium. Write y(t) for both.
Solution:
\[ \omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{\pi}{2} \text{ rad/s} \]
\[ y_1(t) = A \sin(\omega t) \]
\[ y_2(t) = A \sin(\omega t - \Delta\phi) \quad (\Delta\phi = \omega \Delta t) \]
WAVES

Special cases

\[ \Delta \phi = 0 \]
In phase

\[ \Delta \phi = 180^\circ \]
Completely out of phase

Equation of displacement

Special cases:

1. Starting from equilibrium
   \[ y(t) = A \sin(\omega t) \]

2. Starting from maximum:
   \[ y(t) = A \sin(\omega t + 90^\circ) \]
   \[ y(t) = A \cos(\omega t) \]

In general:
\[ y(t) = A \sin(\omega t + \phi) \]
\[ \phi = \text{phase const} \]

**Example:**
Given \[ y = 12 \sin(20\pi t + \pi/6) \]
(a) Find \( A = ? \), \( f = ? \), \( T = ? \)
(b) Where does the object start to move from?

**Example:**
Given \[ y_1 = A \sin(\omega t + \phi_1) \]
\[ y_2 = A \sin(\omega t + \phi_2) \]
Find phase difference

**Definition:**
If phase difference between the sources is constant
\( (\Delta \phi = \text{const}) \) sources are “coherent”.

**Extra:**
Velocity and acceleration formulas in harmonic motion

WAVE MOTION

- Material waves: Water waves, sound waves, earthquake waves.
- Electromagnetic waves: Radio, radar, infrared, light, ultraviolet, gamma…

We will see all these waves share the same common properties:
   a) Reflection, b) refraction (transmission),
c) interference (superposition) and d) diffraction]

Wave motion is propagation of a disturbance through an elastic medium. Particles of medium are not transferred, energy is transferred.

**Sinusoidal (Harmonic) Waves**
[are produced by a source performing simple harmonic motion]

**Definitions**

a) Wavelength (\( \lambda \)): Distance between two crests
Unit: meter

b) Period (T): Time needed for one wavelength to pass a point. Therefore period is the time of production of one wavelength by source.
Unit: second

c) Frequency (f): Number of waves (\( \lambda \)'s) passing a point in one second.
Unit: wave/s, 1/s, Hz (f = 1/T)

d) Amplitude: Maximum displacement from equilibrium.
Unit: meter
I. WAVES ON A SPRING

1. Velocity

Speed of a pulse depends on two factors:

- Force stretching the spring (F)
- Linear density (μ)

(F = μ ; Unit: kg/m)

Formula is: \( v = \sqrt{\frac{F}{\mu}} \)

Therefore: Speed of wave is determined by the properties of the medium. Speed can not be changed from the source.

Therefore: If forces equal

\( v_{\text{light}} > v_{\text{heavy}} \)

Two types of waves:

- Transverse wave: Particle’s motion is perpendicular to wave velocity.

- Longitudinal wave: Particles move parallel to velocity.

Relation among \( v, f, \lambda \):

\( v = \frac{\text{distance}}{\text{time}} \)

\( v = \frac{\lambda f}{f} \quad \lambda = \frac{v}{f} \)

[Analogy: \( \lambda \) is length of one step as you walk, and \( f \) is how many steps you take in one sec.]

Ex: 20 waves are produced by a source in 4 seconds.
(a) Determine \( f = ? \) and \( T = ? \)
(b) Find \( v = ? \) if \( \lambda = 20 \) cm.

Ex: What is the frequency of radio waves having 300 m wavelength?

Ex: 5 wave crests pass through a point in 1 sec. Find \( \lambda = ? \) if \( v = 48 \) cm/s.
Ans: 12 cm

Rule: \( v \) is determined by medium
\( f \) is determined by source
\( \lambda \) is determined by \( \lambda = \frac{v}{f} \)

Amplitude does not have any relation with any of these three.

Energy transmitted by spring waves:

(Power transmitted) \( \sim \) (density) \( \times \) (amplitude)\(^2\) \( \times \) (freq)\(^2\) \( \times \) (vel)

In general (for most sinusoidal waves)

Power \( \sim A^2 f^2 \)

Ex: If wave loses energy due to friction while traveling \( v, f, \lambda \) do not change. Amplitude decreases.

2. Superposition (interference) of spring waves:

\( y = y_1 + y_2 \)

[Waves continue to move passing through each other as if nothing has happened]

Completely out of phase:

Ex: 

Ex:
2. Reflection of spring waves
- From fixed end
- From free end

3. Transmission (refraction) of spring waves
- From light to heavy spring
- From heavy to light spring

{The pulse length in heavy spring is always smaller because frequency does not change during transmission and velocity is smaller. Also in the second picture, if amplitude of incident wave is A, amplitude of transmitted wave is between A and 2A (between fixed and free end reflections). Amplitude of reflected wave is even smaller than A since energy is shared.}

Facts: (During transmission)
1. Frequency does not change [Number of coming waves can not be different from number of transmitted waves]
2. Velocity and wavelength change
3. Transmitted wave never inverted [no phase diff. for transmitted waves]

[Summary: We learnt
a) Superposition (=interference)
b) Reflection
c) Transmission (=Refraction)
of waves on a spring]
Reflection law:

Measuring speed of a wave:
Defn:

\[ v = \frac{\Delta x}{\Delta t} \]

Using stroboscope:

When waves seem to stop:
\[ f_{wave} = N f_{stroboscope} \quad (N=\text{number of slits}) \]
\[ f_{wave} = N f_s \]

[Waves will seem to stop at different frequencies of stroboscope. Why?] We use the highest frequency in the formula. Why]

To find speed we use \( v = f\lambda \).

3. Refraction of Water Waves
Rules:
1. Frequency of waves depend on the source. Frequency does not change when medium changes.
2. Speed of waves depend on medium. In deep water waves are faster, in shallow water waves are slower.
3. Since \( \lambda = \frac{v}{f} \), wavelength changes when medium changes.

Therefore since \( f=\text{const.} \):
\[ \frac{f_d}{f_s} = \frac{v_d}{v_s} = \frac{\lambda_s}{\lambda_d} \]
\[ \uparrow v = f\lambda \uparrow, \quad \downarrow v = f\lambda \downarrow \]

Law of refraction
[In geometrical optics Snell’s law we found starting from \( \frac{v_1}{v_2} = \frac{\sin \theta_1}{\sin \theta_2} \), which we wrote without proof. Now we are verifying it.]

Verify:

\[ \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \frac{\sin \theta_1}{\sin \theta_2} \]

[Correct for any kind of wave]
Note: In Snell’s law we used angle with normal. Now we are using angle with wave front. But:

4. Diffraction
Diffraction is the spreading of a wave passing through a narrow opening.
[Diffraction is a common wave property observed in all kinds of waves, just like all the other properties we are learning for water waves]

Diffraction amount depends on $\frac{\lambda}{a}$ proportion.
If $a >> \lambda$, diffraction is negligible.

[As the second figure shows, diffraction is never zero. Even the smallest deviation from the straight path will be significant after long distances. Therefore: it is impossible to send waves on a straight line over long distances. When we were learning geometrical optics we said “light rays travel on straight paths”. It was an approximation that holds for small distances, and when light does not pass through small (comparable to its wavelength) holes.]

5. Huygens’ principle:
Any point on a wave front of light may be regarded as the source of secondary waves.
[Using this principle, we can explain any wave phenomenon. We will now give three examples]

a) Propagation:

b) Reflection:

[Show that $v_1/v_2 = \sin \theta_1 / \sin \theta_2$]

c) Refraction:

[Show the two-source interference pattern on water surface (or if not available, video or java animation, but an animation will never be as useful as a real ripple tank application)]

Node: The lines on water surface connecting the points where a crest from a source and a trough from the other cancel each other are called “nodal lines”.
Antinode: The lines along which two crests or two troughs meet.

Properties: If sources are in phase,
1. Central line is an antinode
2. Nodes are symmetrical around central line
3. Nodes are numbered as in figure [starting from the one closest to central line.]

Path difference ($\delta$):

$$\delta = |r_1 - r_2|$$
On node n=1:
Path diff. for point A: \( \delta = r_1 - r_2 = 2.5\lambda - 2\lambda = 0.5\lambda \)
Path diff. for point B: \( \delta = r_1 - r_2 = 3.5\lambda - 3\lambda = 0.5\lambda \)

\{Do it for points on n=2 antinode\}
Therefore:
On n=1 node \( \Rightarrow \delta = 0.5\lambda \)
On n=2 node \( \Rightarrow \delta = 1.5\lambda \)
On n=3 node \( \Rightarrow \delta = 2.5\lambda \)
On n=3 node \( \Rightarrow \delta = 3.5\lambda \)

Because:

\[
\begin{align*}
\text{Node} & \quad \delta = \left( \frac{n - 1}{2} \right) \lambda \\
\text{Antinode} & \quad \delta = n \lambda 
\end{align*}
\]

\( n = 1, 2, 3, \ldots \)

[Ex: What is the name of the shape of a node or antinode? \{Parabola, hyperbola, ellipse …\}]

[Ex: Measuring wavelength by using interference pattern]. Point P on the second node is 14 cm and 17 cm away from the sources. Find \( \lambda \). Sources are in phase.

[Ex: Point P is at respective distances 24 cm and 34 cm from the sources. Sources are in phase and producing waves having 4 cm wavelength. Define point P. (On which node or antinode is it?)]

[Ex: Find the distance [s;B]]

[Ex: Draw pattern of nodes and antinodes.]

[Ex: Find speed of water waves, if frequency is given as 15 Hz.]

[Ex: Two sources produce \{not necessarily water\} waves at 50 Hz frequency and 0.7 cm amplitude. Speed of waves in the medium is 100 cm/s. Find the wave amplitude at a point
a) 20 cm and 26 cm to the sources
b) 19 cm and 21 cm to the sources]

7. Phase of a point on the wave
All points on a sinusoidal wave performs simple harmonic motion with same frequency but different phase constant.

\[
y = A \sin(\omega t - \phi)
\]

Point s oscillates with equation \( y_s = A \sin(\omega t) \) Find equation of oscillation of point P.

Solution:

\[
y_p = A \sin(\omega t - \phi) \Rightarrow \phi = \frac{x}{\frac{2\pi}{\lambda}}
\]
Ex: Point P on a long string oscillates as $y = A \sin \left(100\pi t + \frac{\pi}{4}\right)$ as a sinusoidal wave travels to the right direction with speed $v = 10$ m/s. Find equation of oscillation of points a) 15 cm to the right of P b) 35 cm to the right of P b) 15 cm to the left of P.

Hint: $\frac{\Delta \phi}{2 \pi} = \frac{\Delta x}{\lambda}$

8. Phase difference between waves
Phase difference caused by two factors:

a) Path difference ($\delta$):
Sources start together but distances are different:

$$s_1 \text{ and } s_2 \text{ are moving together (in phase), but } P_1 \text{ and } P_2 \text{ are not.}$$

Since $1 \lambda \Leftrightarrow 2 \pi$

$$\frac{\Delta \phi}{2 \pi} = \frac{\delta}{\lambda}$$

Ex: Special cases:
For all points on:

Node n=1 ($\delta = 0.5 \lambda \Rightarrow \Delta \phi = \pi$)

Node n=2 ($\delta = 1.5 \lambda \Rightarrow \Delta \phi = 3\pi$)

Node n=3 ($\delta = 2.5 \lambda \Rightarrow \Delta \phi = 5\pi$)

Antinode n=0 ($\delta = 0 \Rightarrow \Delta \phi = 0$)

Antinode n=1 ($\delta = 1 \lambda \Rightarrow \Delta \phi = 2\pi$)

Antinode n=2 ($\delta = 2 \lambda \Rightarrow \Delta \phi = 4\pi$)

b) Time delay ($\Delta t$): Distances equal but one source starts $\Delta t$ later.

In general:
Since $1 \lambda \Leftrightarrow 1 T \Leftrightarrow 2 \pi$

$$\frac{\Delta \phi}{2 \pi} \Leftrightarrow \frac{\text{Path difference}}{\text{Wavelength}} \Leftrightarrow \frac{\text{Time difference}}{\text{Period}}$$

$$\Delta \phi = \frac{\delta}{\lambda} \text{ and } \Delta \phi = \frac{\Delta t}{T}$$

Ex: Write node and antinode formulas if one source starts $\Delta t$ later.

Sol: Now we have both time and path delay. Therefore we add the effects:

$$\frac{\Delta \phi}{2 \pi} = \frac{\delta}{\lambda} + \frac{\Delta t}{T}$$

[For all points on n=1 node $\Delta \phi = \pi$]

For all points on n=2 node $\Delta \phi = 3\pi$]

For all points on n=3 node $\Delta \phi = 5\pi$]

For all points on n'th node $\Delta \phi = (2n-1)\pi$

$$\frac{(2n-1)\pi}{\pi\lambda} = \frac{\delta}{\lambda} + \frac{\Delta t}{T}$$

$$\delta = n - 1 \frac{\Delta t}{T}$$

$$\Delta \phi = \left(n - 1 \frac{\Delta t}{T}\right) \lambda$$

(n = 1, 2, 3, 4…)

In general:
If one starts $\Delta t$ later (or earlier):

n'th node:

$$\delta = \left(n - 1 \frac{\Delta t}{T}\right) \lambda$$

n'th antinode:

$$\delta = \left(n - 1 \frac{\Delta t}{T}\right) \lambda$$

{± in formula is about starting earlier or later. But since there are two sources and we choose one of them as the reference (starting at $t=0$) arbitrarily, we can always use + in the formula}

Special case:
If $\Delta t = \frac{T}{2} \Rightarrow (\Delta \phi = 180^\circ) \Rightarrow$ sources completely out of phase, nodes and antinodes change place. Central line becomes a node.

Ex: Two sources wave sources have the same period 0.4 s. One of the sources start 0.1 s. later. Find phase difference between the sources.

Ex: Two sources on water surface produce waves of same frequency. ($\lambda = 4$ cm). Find phase difference of waves at a point 25 cm and 24 cm away from the waves if sources start together (sources in phase).

Ex: Point P is 17 cm and 14 cm away from the sources. Find phase difference of waves at point P if $\lambda = 0.6$ cm. Sources in phase.

Ex: a) Two sources wave sources have the same period 0.4 s. One of the sources start 0.1 s. later. Find phase difference between the sources.

b) Find phase difference between waves at a point 23.8 cm and 24.3 cm away from the sources. ($\lambda = 3$ cm)
III. SOUND WAVES

STANDING WAVES