WAVE OPTICS

1. Wave Theory of Light

(by Huygens, Fresnel, Young, etc…)

- In geometric optics we learnt light is a stream of straight-going particles (Newton proposed that first)
- Then we learnt light is a form of EM wave.
- But we had learnt all waves have common characteristic properties such as:

Reflection, refraction, interference, diffraction…

- Therefore light waves must have all these properties. Now we will learn:

* Reflection,
* Refraction,
* Dispersion,
* Interference,
* Diffraction and
* Polarization of light waves.

- Actually Huygens had already said light was a form of wave motion, long before Maxwell speculated about EM waves.

2. Properties of Light Waves

- Light waves are transverse [we already know this from EM waves]
- Amplitude of light wave can mean amplitude of electric or magnetic field component, because they are always proportional (E=cB) [But when we speak about amplitude of light waves we generally have electric field component in mind. This is because most of the optical phenomena are caused by this component]
- Color of light is determined by frequency (or wavelength) light waves.

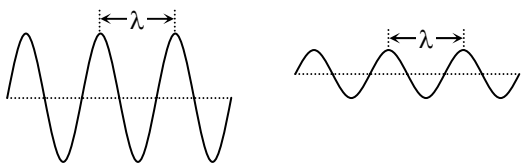
red	orange	yellow	green	blue	violet
←				→	
lower frequency				higher frequency	
longer λ				shorter λ	

- Brightness of a light wave is determined by amplitude of light wave.

Brightness ~ $(\text{Amplitude})^2$

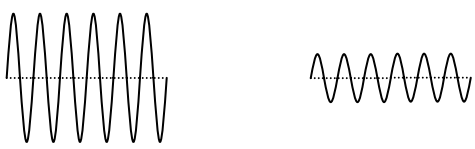
So:

Bright red light: Dim red light:



f = f

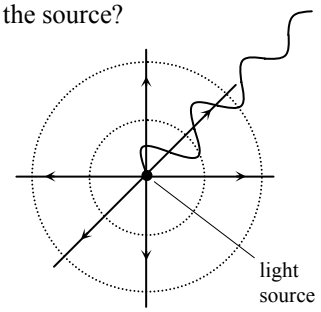
Bright blue light: Dim blue light:



f = f

Ex: [We know that if spring wave loses energy due to friction while traveling v, f , λ do not change. Amplitude decreases.]

For EM wave there is no friction. So why is the brightness (amplitude) decreasing as we go away from the source?



Note: Frequency, wavelength and speed of light waves do not change as they propagate away from the source. Only amplitude decreases.

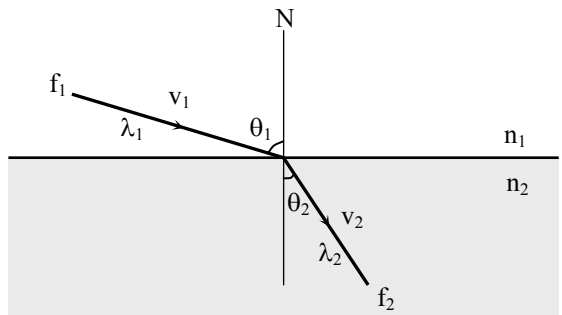
[Otherwise a blue light source would be observed as red from far away]

Ex: Can we say "intensity" in place of "brightness"?

{Remind definition and unit of intensity if needed. Also remind energy transmitted by a wave on a coil spring was proportional to amplitude squared}

Ex: Find the relation between the intensity of light and distance from the source.

3. Refraction of Light Waves



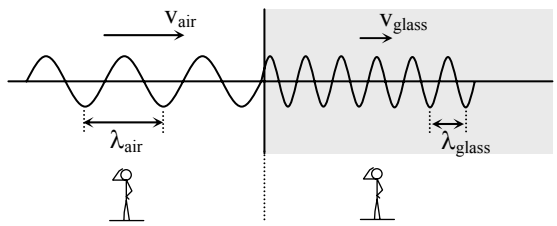
$$\frac{Sin\theta_1}{Sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = n_{12}$$

Rule: When a wave changes medium,

a) Frequency does not change

b) Speed changes

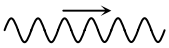
Therefore:



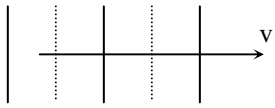
Both observers count the same number of wave crests in one second.

Special case: If light is coming from air

$\lambda_{glass} = \dfrac{\lambda_{air}}{n_{glass}}$ because $n_{air}=1$

Caution: Drawing this figure  for a light wave does NOT mean that light rays move up and down in the air. [The figure is trying to say that electric field at a point is increasing and decreasing (oscillating) as the light passes by. This oscillation itself is called light.

Remember: In water waves,



each water molecule is moving up and down as a crest or trough passes by. But we do not say the water wave is following a sinusoidal path. When we are asked to draw the path of the wave, we draw a straight arrow showing direction of motion in general, not motion of particles. And since the wave is transverse, direction of motion is perpendicular to up-down motion of particles.]

4. Dispersion of Light Waves

Dispersion means dependence of index of refraction of a medium on the frequency (or wavelength) of the incident light.

That is, for example:

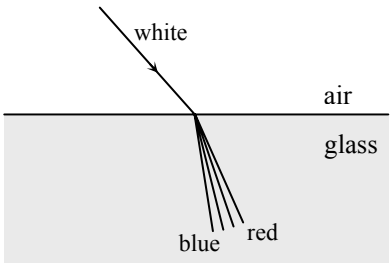
$n_{glass} \neq 1.5 = \text{constant}$

$n_{glass} = n(\lambda)$

for red light $n_{glass} = 1.513$

for blue light $n_{glass} = 1.528$

Therefore:



5. Interference of Light Waves

Coherence:

If two wave sources are "coherent" they always have the same phase difference between them. [If they are in phase at the beginning, they are always in phase. If they start 180° out of phase they will still be 180° out of phase 10 minutes later]

[If two waves are coherent at a point in space, they always have the same phase difference at this point in space]

- If two wave sources are coherent, the interference pattern is stable and observable. [Nodes and antinodes will always be at the same place, we will be able to see them]
- If two wave sources are incoherent, the interference pattern is not observable. [Think about central line in ripple tank. Suppose now the sources are in phase, and central line is an antinode, (say) 0.23 second later sources become completely out of phase, central line becomes a node, (say) 0.36 second later they become in phase again and central line is an antinode. So we will not see any interference pattern]

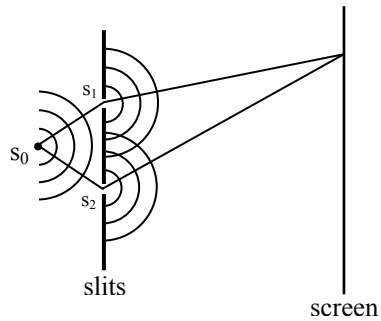
Question: [When we have two wave sources on water, we see several nodes on water surface where waves from two sources cancel]. Why don't we ever see light waves from two lamps cancel each other and some points in the room become dark (= node)?

Answer: {Explain the reason, why two light bulbs (or any other ordinary light sources) can never be coherent, then ask the students to find a way for obtaining two coherent light sources}

5.a. Young's Experiment

To obtain two coherent light sources Thomas Young used one single light source and made the light pass through two slits (slit = a long narrow opening). Now each slit is like a light source.

Note: To obtain a better visible interference pattern sources must be monochromatic (of one color = having single λ).



On the screen we see: We simply draw:

Note: These dark and bright bands are called "fringes".

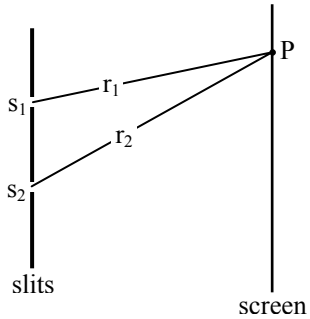
Remember: Node for water ⇒ Dark for light

Antinode for water ⇒ Bright for light

[Actually there are no definite boundaries between dark and bright fringes. Only the center of a dark fringe is totally dark and center of a bright fringe is maximum bright]

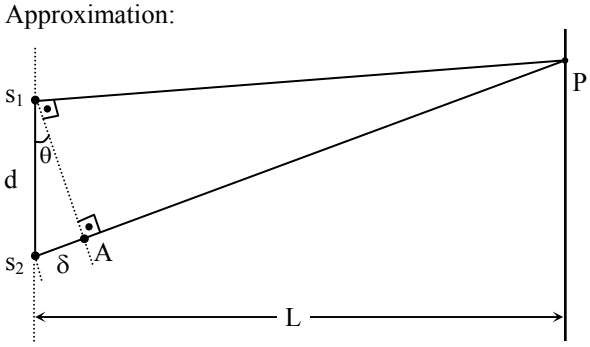
Class demo: Hold a thin glass plate over the flame of a candle. When it is black enough draw two slits with a razor blade. Illuminate the slits with a laser beam. Use a white paper 1 meter away as the screen.

Path difference (δ):



$$\delta \equiv |r_1 - r_2|$$

Finding δ from geometry:
Approximation:



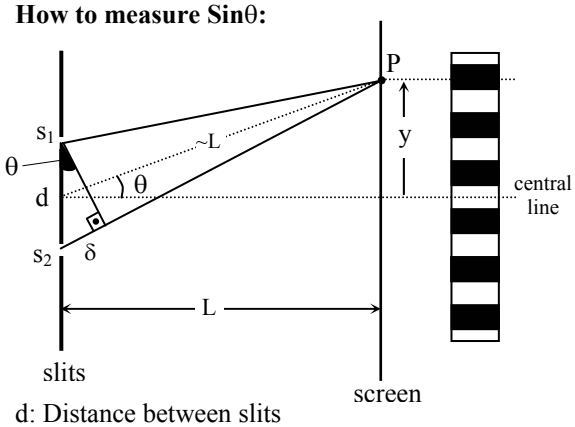
[d: Distance between sources
L: slits-screen distance]
Since L ~ 1 meter
d ~ 0.1 mm;
L>>>d so we can take $s_1P \parallel s_2P$

therefore: $|s_1P| \approx |AP|$
therefore: $|s_2A| = |r_1 - r_2| = \delta$

therefore: $\boxed{\delta = d\ Sin\theta}$

[This formula seems to be totally useless, because we can not even see θ let alone measuring it. But:]

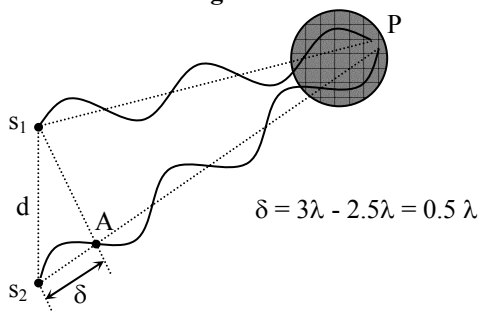
How to measure Sinθ:

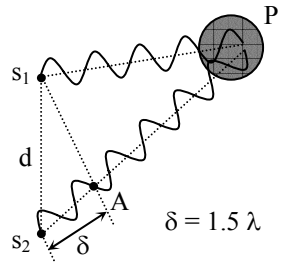


d: Distance between slits
L: Slits-screen distance
y: Distance from central line to a point on a fringe

$$\boxed{\delta = d\ Sin\theta} \Leftrightarrow \boxed{\delta = d\ \frac{y}{L}}$$

Condition for dark-bright:





dark	bright
δ = 0.5 λ	δ = 0
δ = 1.5 λ	δ = 1 λ
δ = 2.5 λ	δ = 2 λ

Dark

$$\delta = \left(m - \frac{1}{2}\right)\lambda$$

$$d\ Sin\theta = \left(m - \frac{1}{2}\right)\lambda$$

$$d\frac{y}{L} = \left(m - \frac{1}{2}\right)\lambda$$

$$\boxed{y_m = \left(\frac{\lambda L}{d}\right)\left(m - \frac{1}{2}\right)}$$

$$m = 1, 2, 3...$$

Bright

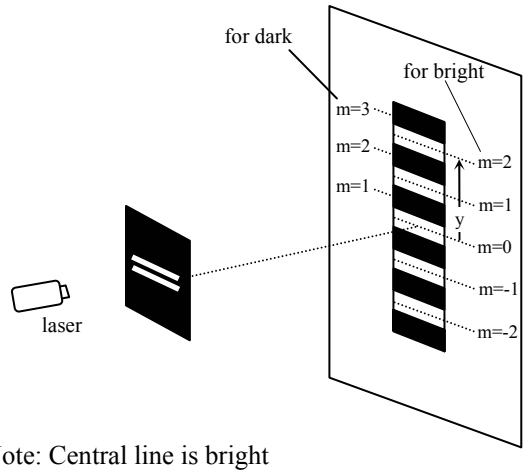
$$\delta = m\lambda$$

$$d\ Sin\theta = m\lambda$$

$$d\frac{y}{L} = m\lambda$$

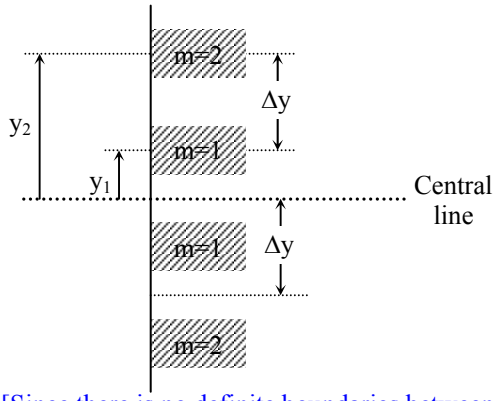
$$\boxed{y_m = \left(\frac{\lambda L}{d}\right)m}$$

$$m = \underbrace{0}_{central\ line}, 1, 2, 3...$$



Note: Central line is bright
Note: We have m=2 for dark, m=2 for bright
Note: y starts from central line
Note: m is always integer

Fringe width (Δy):



[Since there is no definite boundaries between dark and bright fringes, we take the region between two absolute darks (at the center of the dark fringe) as the width of a bright fringe.]

$\Delta y = y_2 - y_1$

$$\Delta y = \left[\frac{\lambda L}{d}\left(2-\frac{1}{2}\right)\right] - \left[\frac{\lambda L}{d}\left(1-\frac{1}{2}\right)\right]$$

$$\boxed{\Delta y = \frac{\lambda L}{d}}$$

Note: Units used for λ.
1 μm = 10^{-6} m (micrometer)
1 nm = 10^{-9} m (nanometer)
1 Å = 10^{-10} m (angstrom)

Ex:

Color	A	nm	m
red	6000	600	6×10^{-7}
blue	4000	400	4×10^{-7}

Ex: 6000 A laser light passes through two slits 0.1 mm apart and reaches the screen placed 2 m away.
a) Find fringe width
b) Find position of second dark
c) Find position of third bright
{Draw figure after solution}

Ex: Laser light (5000 A) passes through a double slit arrangement 0.05 mm apart. The screen is 1 m away from slits.
a) Find fringe separation (=fringe width)
b) Find distance between 2nd bright and 3rd dark on opposite sides.
{Draw figure during solution}

Ex: What can we do to obtain a better visible pattern in Young's experiment?
{Explain effect of changing λ, d and L on Δy. Draw two example patterns for small and large d}

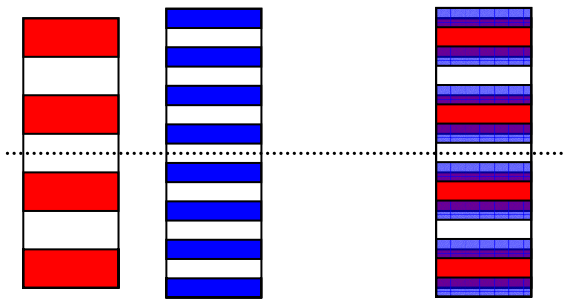
[Result: Slits closer, fringe centers distant. Slits distant, fringe centers closer]

Ex: Suppose while performing double-slit experiment, the space between the slits and the screen is filled with water. How does the interference pattern change?

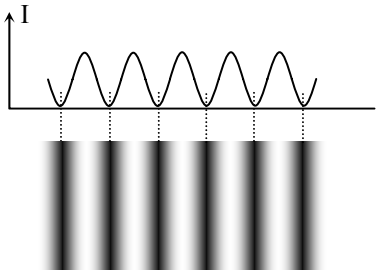
Ex: A double-slit arrangement is illuminated first with red, then with blue light.
a) Which one has wider fringes?
b) Which one produces more fringes?

Ex: What happens if we use white light in place of monochromatic light in Young's experiment?
Answer: Think about light of two color only (red-blue)

only red: only blue: together:



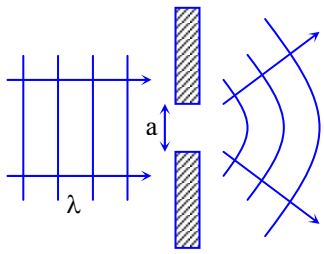
Intensity Distribution:



Rule: In double-slit interference, all fringes are equally bright and wide. {Actually we are neglecting diffraction effects for the time being. We take the slits sufficiently small themselves so as to make diffraction effects negligible. See N-slit diffraction}

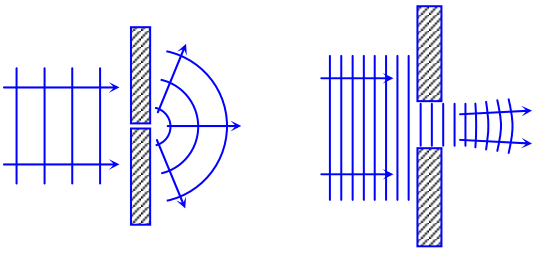
6. Diffraction
Diffraction is bending of waves around an obstacle (barrier) [or spreading of waves passing through a narrow slit]

[We had seen diffraction with water waves



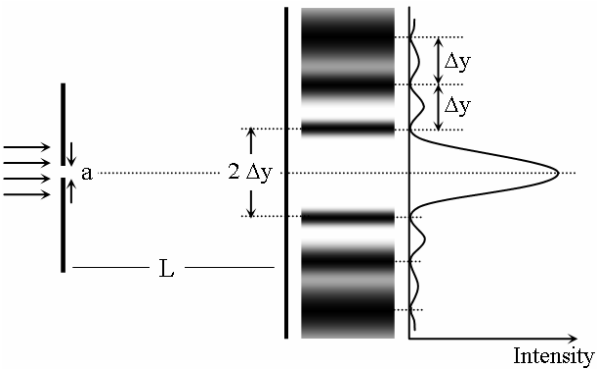
Diffraction amount depends on $\frac{\lambda}{a}$ proportion.

If a>>λ diffraction is negligible.



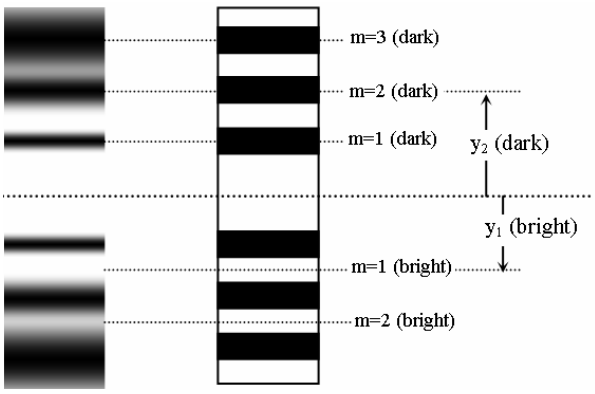
Same phenomenon is observable with light waves. Since λ of light is very small, the openning must also be very small, something like 0.1 mm]

Single slit diffraction:



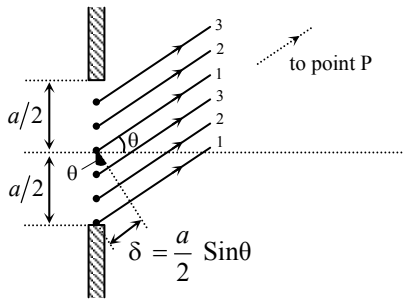
[Most of the light energy is concentrated at the central maximum. Actually it is possible to say that all the light passing through the slit is spread as wide as the central maximum simply omitting the other bright fringes]

Actual pattern: We simply draw:



[We still have dark fringes althought there is only one slit. Therefore light waves coming from different portions of the slit must be canceling]

If we divide the slit into two equal portions:



Now condition for first dark:

$$\delta = \frac{\lambda}{2} \Rightarrow \frac{a}{2} Sin\theta = \frac{\lambda}{2}$$

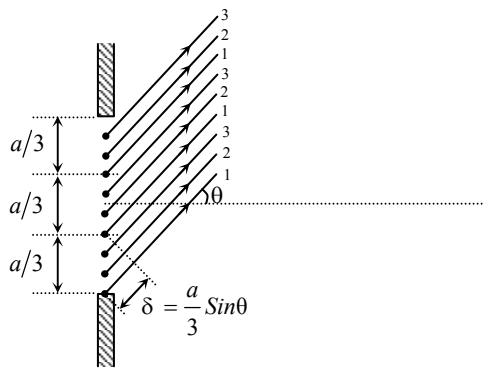
$$\boxed{a\ Sin\ \theta = \lambda \text{ (first dark)}}$$

[First dark is important, because between two first darks we have the central bright, which receives nearly all the light energy passing through the slit]
if we divide the slit into 4, 6, 8, (even number) equal parts [and set δ=λ/2 we will have (a/4)Sinθ=λ/2, (a/6)Sinθ=λ/2, (a/8)Sinθ=λ/2…] we get condition for other darks {explain relation between even number and dark}:

Dark	Bright
a Sinθ = mλ	a Sinθ=(m+1/2)λ
$a\frac{y_m}{L} = m\lambda$	$a\frac{y_m}{L} = \left(m+\frac{1}{2}\right)\lambda$
$\boxed{y_m = \frac{\lambda L}{a} m}$	$\boxed{y_m = \frac{\lambda L}{a}\left(m+\frac{1}{2}\right)}$
m=1,2,3…	m=1,2,3…

[We don't have m=0 for central bright. Central bright is determined by position of first darks]

Ex: How have we found condition for brights?
Sol: Divide the slit into 3, 5, 7, (odd number) parts: For first bright (m=1) we divide the slit into 3 equal portions. [Because "dividing" into 1portion gives us the central bright]



For first bright (m=1) $\delta = \frac{a}{3} Sin\theta = \frac{\lambda}{2}$ waves from two portions cancel but the remaining third portion illuminates the point on screen. So for first bright

$$m = 1 \Rightarrow a\ Sin\theta = \frac{3}{2}\lambda = \left(m+\frac{1}{2}\right)\lambda$$

Ex: Derive fringe seperation formula Δy=?

$\Delta y = \frac{\lambda L}{a}$ [same between centers of brights and darks, only central bright 2Δy]

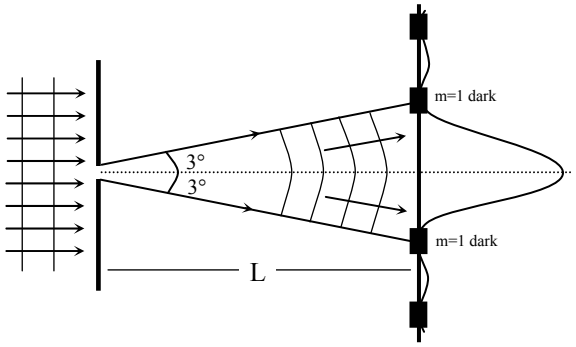
Ex: 5000 A monochromatic light passes through a slit having 0.05 mm width. How much does it spread?

Sol: θ_1 for first dark:
a Sinθ=mλ

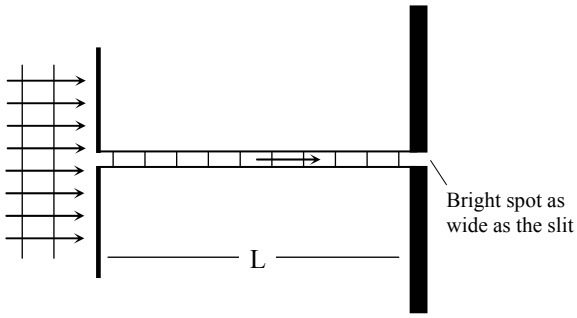
m=1⇒ a Sinθ = λ

$$\boxed{\text{Sin}\theta_1(dark) = \frac{\lambda}{a}}$$

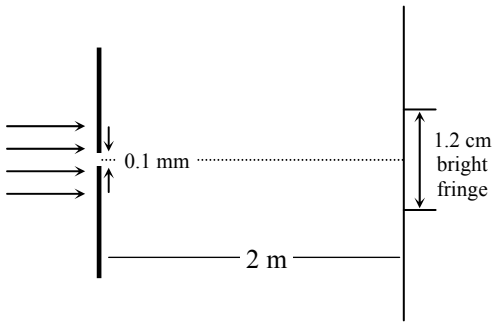
$\text{Sin}\theta_1 = 5 \times 10^{-7} / 10^{-5} = 0.05$
$\theta_1 \approx 3°$



If there wasn't diffraction:



Ex: Monochromatic light (λ=6000 A) passes through a slit 0.1 mm wide and illuminates a screen 2 m away. Find width of central bright on screen.
Answer:
2Δy= 12 mm =1.2 cm



If there wasn't diffractionthere would be a bright spot 0.1 mm wide on the screen.

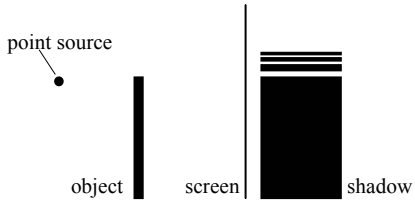
Ex: Laser light having 6000 A wavelength passes through a slit 0.2 mm wide. On a screen placed 1 m away find
a) Distance from central line to second bright
b) Distance between second dark and third bright on different sides.

Ex: What are the effects of diffraction?
* We can not send a light ray along a straight path for a long distance. It will spread and lose intensity.
[Actually this is the case for any type of EM wave]

[Imagine otherwise, we would be able to send mors code messages to an astronout on the moon by using a simple diode laser.]

* We don't have sharp shadows of objects even with a point light source.

Ex: Diffraction from an edge (not a slit)

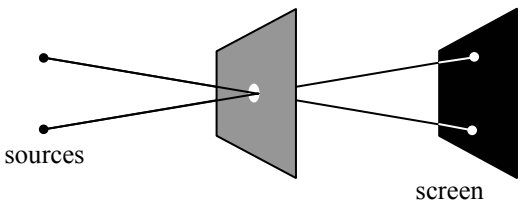


Ex: What is the minimum slit width for no diffraction minimum (dark fringe) to be observed?

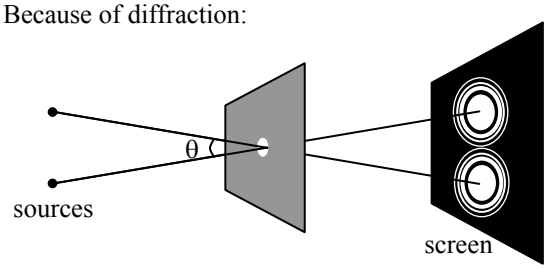
Note: Boundary between geometric optics and wave optics:
There is no definite limit. Depends on:
- Width of light source
- Distance light travels
{Explain using the example below}

Ex: What is the maximum slit width for diffraction?
[Answer: If the light source is coherent, diffraction always occurs for all openings even if the slit is large. But according to formula: $Sin\ \theta = \frac{\lambda}{a}$ (first dark), if a>>λ, then θ is very small. So diffraction effect becomes negligible over small distances. The light follows nearly a straight path as wide as the slit for small distances if the slit dimension is large. But over large distances a small angle causes a a large seperation. If we are trying to send a 5 mm wide laser ray from earth to moon for example, the spreading of the beam will be ~ 0.01°, which is negligible at the beginning. But when it reaches the moon, the beam will be as wide as ~ 80 km!.]

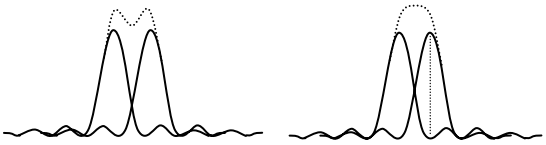
7. Resolving Power
Two light sources are seen as a single source if they are far away enaugh. [Many bright dots in the night sky are actually star pairs – not sinle stars. Enother example can be the two headlights of a car approaching from a distance]
The reason is diffraction. When light from the sources passes through the pupil of the eye, [which is a circular opening of ~2-3 mm] diffraction occurs. The retina acts as a screen.
If there was not diffraction:



Because of diffraction:

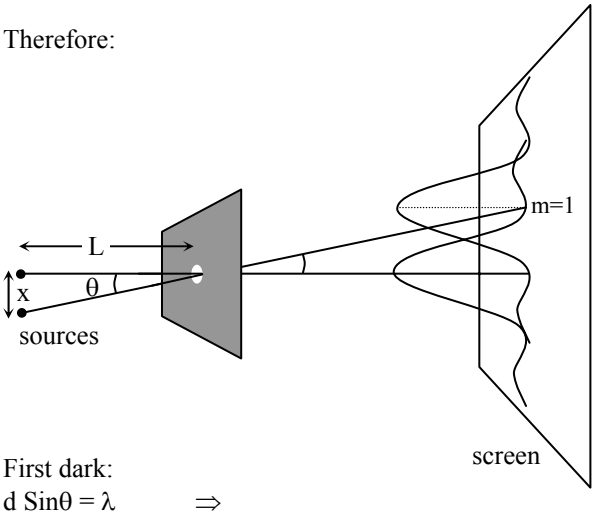


When θ gets smaller patterns overlap and seen as one:



Rule: Two sources seen as one when central bright of one pattern is on the first dark of the other.

Therefore:



First dark:
d Sinθ = λ ⇒

$Sin\theta = \frac{\lambda}{a}$ $\frac{x}{L} > \frac{\lambda}{a}$ two sources seen

$\frac{x}{L} = \frac{\lambda}{a}$ (just resolved) $\frac{x}{L} < \frac{\lambda}{a}$ seen as one source

{Actually these formulas are for slits, and can be used for a cat for example. For circular apertures we have a factor of 1.22 which we neglected here}

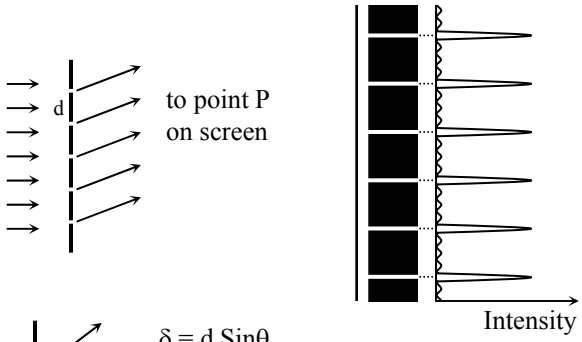
Ex: From what distance can we see two headlights of a car as two?
Distance between lights 1.5 m, take λ=5000A.
[The actual distance is much smaller due to other (such as atmosheric conditions. Diffraction is the ultimate limit in our 'seeing' power and since there are always other factors limiting our vision we are seldom limited by diffraction effects]

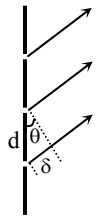
[**Ex:** Explain whys we can't ever see an atom with normal light no matter how powerful a microscope we use. That is, explain how diffraction puts a limit to seeing small objects]

Ex: Explain why very large dishes are used for radiotelescopes.

8. Diffraction Grating

{Demo: N-slit diffraction java applet}
[The diffraction grating is a more useful device to analyse lights sources, because the interference maximums (bright fringes) are thin lines, making the measurements easier]





δ = d Sinθ
Therefore; m'th BRIGHT fringe:
$\boxed{d\ Sin\theta = m\ \lambda}$ (m = 0, 1, 2, 3 …)

[Therefore we can use two slit formulas]
[We are not writing formula for dark fringe because dark friges are actullly wide dark bands between two bright lines]

Ex: A diffraction grating has 500 slits in 1 cm.
a) Find slit spacing
b) Find λ of monochromatic light if first maximum (bright fringe) occurs 3.5 cm from the central line on a screen 1 m away.

Ex: Monochromatic light of 650 nm wavelength is incident on diffraction grating having $2x10^{-6}$ m slit spacing.
a) How many bright lines will be observed?
b) What is the angular position of the first diffraction fringe ?

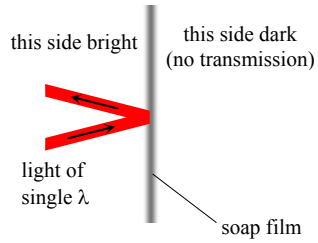
Ex: What is the path difference for the light waves forming the bright fringe at 30° from the central bright? Slit spacing of the diffraction grating is 0.05 mm.

Ex: A diffraction grating is illuminated by mixed red and blue light. Second bright of red coincides with the third bright of blue. Find λ_{blue}=?, if λ_{red}= 6000 A.

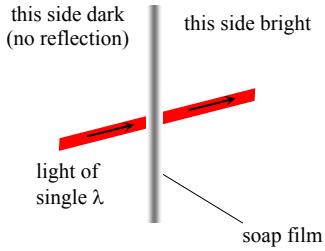
Ex: How many slits in 1 mm must a diffraction grating have, if it is to be used to analyse light having wavelength around 0.5 μm?

9. Interference in thin films

Extreme case 1:



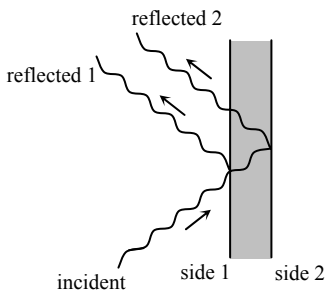
Extreme case 2:



In intermediate cases light is partly transmitted, partly reflected.

{Ask students: soap bubble is normally transparent, how can it stop light. Why soap bubble? Is it because it is very thin?}

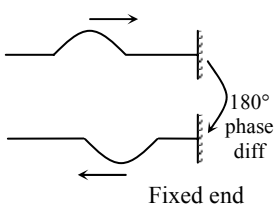
A soap bubble has two sides:

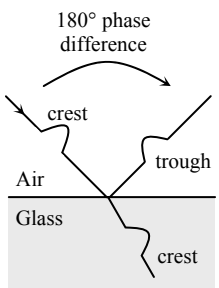


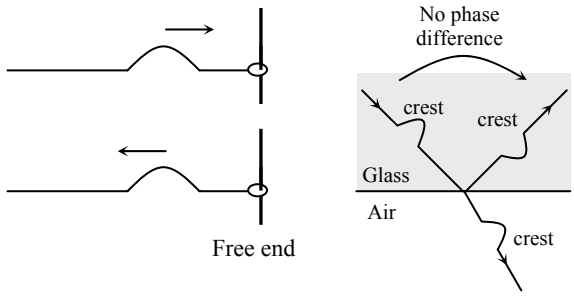
Light rays reflecting from two sides can cancel or reinforce according to phase difference between them.

Rule:

Remember waves on a spring. Light waves have the same property.



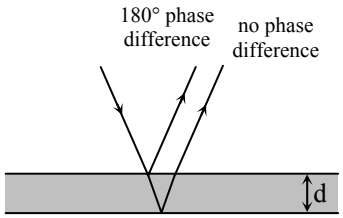




[Rule: light rays undergo 180° phase change upon reflection from an optically denser (with greater index of refraction) medium.]

Formula:

For observer looking from above:



[So between two reflected rays there is 180° phase diff]

$$180^\circ \text{ phase difference} \Leftrightarrow \frac{\lambda}{2} \text{ path difference}$$

For destructive interference (dark):

$$\text{Path difference} = (m - \frac{1}{2})\lambda_{film} \qquad \boxed{\lambda_{film} = \frac{\lambda_{air}}{n_{film}}}$$

{Explain 2d}

$$\underbrace{2\text{d} \pm \frac{\lambda_{film}}{2}}_{\text{equivalent path difference}} = (m - \frac{1}{2})\lambda_{film}$$

[We can add or subtract λ/2. This just means we take one wave as being λ/2 in front of or behind the other, which is not important because the situation is symmetrical. We will use the minus sign, because when we use the minus sign, we can start from m=0 ⇒ zero thickness. Otherwise we would start from m = -1,which is possible but not very nice. Remember m is just a counting number, 1st order dark, 2nd order dark etc.]

Looking from above:

[Looking from above means the light source and the observer are on the same side of the soap film]

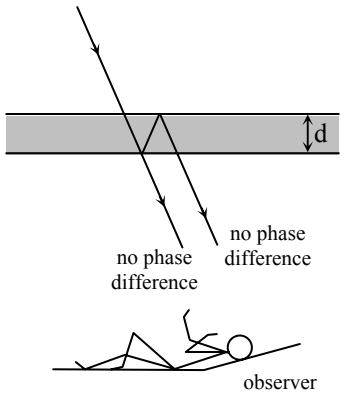
$$\boxed{2d = m\lambda_{film} \quad \Leftrightarrow \quad \text{Dark}} \quad m = 0, 1, 2, 3, 4\ldots$$

Therefore:

$$\boxed{2d = \left(m - \frac{1}{2}\right)\lambda_{film} \quad \Leftrightarrow \quad \text{Bright}} \quad m = 1, 2, 3, 4\ldots$$

[m=0 ⇒ zero thickness. How can this happen? We will see in a minute]

For observer looking from below:



For destructive interference (dark):

$$\text{Path difference} = (m - \frac{1}{2})\lambda_{film}$$

$$\boxed{2d = \left(m - \frac{1}{2}\right)\lambda_{film} \quad \Leftrightarrow \quad \text{Dark}} \quad m = 1, 2, 3, 4\ldots$$

Therefore:

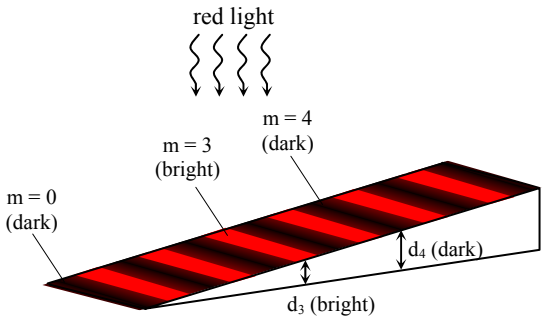
$$\boxed{2d = m\lambda_{film} \quad \Leftrightarrow \quad \text{Bright}} \quad m = 0, 1, 2, 3, 4\ldots$$

Formulas changed place
Therefore:
One side dark ⇔ other side bright.

Ex: 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to be able to pass to other side.
Ex: 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to reflect back from the film surface.

Ex: 6000 A laser light is incident on a soap film (n=1.5). Find three different thicknesses the film might have, if the light is not reflecting back.

Ex: Film of changing thickness. [If you hold a soap film vertically lower side becomes thicker.]



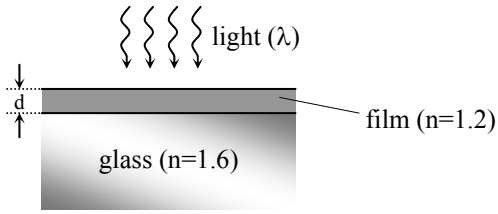
Ex: Explain why we see many different colors over a soap film.

Ex: Lenses used in a camera are generally coated with a thin film of definite thickness. Why?

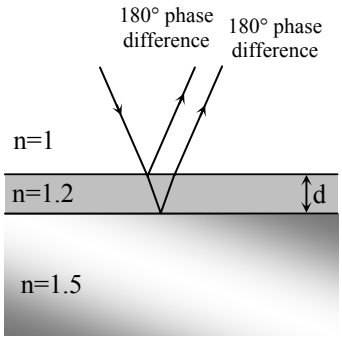
{Film thickness is adjusted to wavelength of yellow light, since it is the most intense component of sunlight}

Ex: Solar cells are also coated with thin films. Why?

Ex: **Thin coating.** Find the formula for thickness of film, if no light of wavelength λ is to reflect back.



Solution:



Dark:

$$2d = \left(m - \frac{1}{2}\right)\lambda$$

$$m = 1, 2, 3\ldots$$

10. Air Wedge