WAVE OPTICS

1. Wave Theory of Light
(by Huygens, Fresnel, Young, etc…)
• In geometric optics we learnt light is a stream of straight-going particles (Newton proposed that first)
• Then we learnt light is a form of EM wave.
• But we had learnt all waves have common characteristic properties such as:
  • Reflection, refraction, interference, diffraction…
  • Therefore light waves must have all these properties. Now we will learn:
    * Reflection,
    * Refraction,
    * Dispersion,
    * Interference,
    * Diffraction and
    * Polarization of light waves.
• Actually Huygens had already said light was a form of wave motion, long before Maxwell speculated about EM waves.

2. Properties of Light Waves
• Light waves are transverse [we already know this from EM waves]
• Amplitude of light wave can mean amplitude of electric or magnetic field component, because they are always proportional (E=cB) [But when we speak about amplitude of light waves we generally have electric field component in mind. This is because most of the optical phenomena are caused by this component]
• Color of light is determined by frequency (or wavelength) light waves.

- Brightness of a light wave is determined by amplitude of light wave.
  Brightness ~ (Amplitude)^2

So:
Bright red light: Dim red light:
\[ \frac{\lambda_1}{f} = \frac{\lambda_2}{f} \]
Bright blue light: Dim blue light:
\[ \frac{\lambda_1}{f} = \frac{\lambda_2}{f} \]

Ex: [We know that if spring wave loses energy due to friction while traveling \( v, f, \lambda \) do not change. Amplitude decreases.]
For EM wave there is no friction. So why is the brightness (amplitude) decreasing as we go away from the source?

Note: Frequency, wavelength and speed of light waves do not change as they propagate away from the source.
Only amplitude decreases.
[Otherwise a blue light source would be observed as red from far away]

Ex: Can we say “intensity” in place of “brightness”?
{Remind definition and unit of intensity if needed.
Also remind energy transmitted by a wave on a coil spring was proportional to amplitude squared}

Ex: Find the relation between the intensity of light and distance from the source.

3. Refraction of Light Waves

Ex: When a wave changes medium,
a) Frequency does not change
b) Speed changes
Therefore:

\[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = n_\text{glass} \]

Both observers count the same number of wave crests in one second.
Special case: If light is coming from air
\[ \lambda_{\text{glass}} = \frac{\lambda_{\text{air}}}{n_{\text{glass}}} \quad \text{because } n_{\text{air}} = 1 \]

Caution: Drawing this figure for a light wave does NOT mean that light rays move up and down in the air. [The figure is trying to say that electric field at a point is increasing and decreasing (oscillating) as the light passes by. This oscillation itself is called light.]

Remember: In water waves,

\[
\begin{array}{c}
\text{v} \\
\text{each water molecule is moving up and down as a crest or trough passes by. But we do not say the water wave is following a sinusoidal path. When we are asked to draw the path of the wave, we draw a straight arrow showing direction of motion in general, not motion of particles. And since the wave is transverse, direction of motion is perpendicular to up-down motion of particles.} \\
\end{array}
\]

4. Dispersion of Light Waves
Dispersion means dependence of index of refraction of a medium on the frequency (or wavelength) of the incident light.
That is, for example:
\[ n_{\text{glass}} \neq 1.5 = \text{constant} \]
\[ n_{\text{glass}} = n(\lambda) \]
for red light \( n_{\text{glass}} = 1.513 \)
for blue light \( n_{\text{glass}} = 1.528 \)
Therefore:

5. Interference of Light Waves

Coherence:
If two wave sources are "coherent" they always have the same phase difference between them. [If they are in phase at the beginning, they are always in phase. If they start 180° out of phase they will still be 180° out of phase 10 minutes later]
[If two waves are coherent at a point in space, they always have the same phase difference at this point in space]

- If two wave sources are coherent, the interference pattern is stable and observable. [Nodes and antinodes will always be at the same place, we will be able to see them]
- If two wave sources are incoherent, the interference pattern is not observable. [Think about central line in ripple tank. Suppose now the sources are in phase, and central line is an antinode, (say) 0.23 second later sources become completely out of phase, central line becomes a node, (say) 0.36 second later they become in phase again and central line is an antinode. So we will not see any interference pattern]

Question: [When we have two wave sources on water, we see several nodes on water surface where waves from two sources cancel]. Why don’t we ever see light waves from two lamps cancel each other and some points in the room become dark (= node)?
Answer: {Explain the reason, why two light bulbs (or any other ordinary light sources) can never be coherent, then ask the students to find a way for obtaining two coherent light sources}
Path difference ($\delta$):

$$\delta = |r_1 - r_2|$$

Finding $\delta$ from geometry:

Approximation:

$$\delta = d \sin \theta$$

[d: Distance between sources
L: slits-screen distance]
Since $L \sim 1$ meter
d $\sim 0.1$ mm;
$L\gg d$ so we can take $s_1P \parallel s_2P$
therefore: $|s_1P| \approx |AP|
therefore: $|s_2A| = |r_1 - r_1| = \delta$

therefore: $\delta = d \sin \theta$

[This formula seems to be totally useless, because we can not even see $\theta$ let alone measuring it. But:]

How to measure $\sin \theta$:

$$\delta = d \sin \theta$$

d: Distance between slits
L: Slits-screen distance
y: Distance from central line to a point on a fringe

Condition for dark-bright:

<table>
<thead>
<tr>
<th>dark</th>
<th>bright</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0.5 \lambda$</td>
<td>$\delta = 0$</td>
</tr>
<tr>
<td>$\delta = 1.5 \lambda$</td>
<td>$\delta = 1 \lambda$</td>
</tr>
<tr>
<td>$\delta = 2.5 \lambda$</td>
<td>$\delta = 2 \lambda$</td>
</tr>
</tbody>
</table>

Dark

$$\delta = \left( m - \frac{1}{2} \right) \lambda$$

$$d \sin \theta = \left( m - \frac{1}{2} \right) \lambda$$

$$d \frac{y}{L} = \left( m - \frac{1}{2} \right) \lambda$$

$$y_m = \left( \frac{\lambda L}{d} \right) \left( m - \frac{1}{2} \right)$$

$$m = 1, 2, 3, ...$$

Bright

$$\delta = ml \lambda$$

$$d \sin \theta = ml \lambda$$

$$d \frac{y}{L} = ml \lambda$$

$$y_m = \left( \frac{\lambda L}{d} \right) m$$

$$m = 0, 1, 2, 3, ...$$

Note: Central line is bright
Note: We have $m=2$ for dark, $m=2$ for bright
Note: $y$ starts from central line
Note: $m$ is always integer
Fringe width ($\Delta y$):

Since there is no definite boundaries between dark and bright fringes, we take the region between two absolute darks (at the center of the dark fringe) as the width of a bright fringe.

$$\Delta y = y_2 - y_1$$

$$\Delta y = \frac{\lambda L}{d} \left( 2 - \frac{1}{2} \right) - \left[ \frac{\lambda L}{d} \left( 1 - \frac{1}{2} \right) \right]$$

Note: Units used for $\lambda$.

1 $\mu$m = $10^{-6}$ m (micrometer)
1 nm = $10^{-9}$ m (nanometer)
1 Å = $10^{-10}$ m (angstrom)

Ex: 6000 Å laser light passes through two slits 0.1 mm apart and reaches the screen placed 2 m away.

a) Find fringe width
b) Find position of second dark
c) Find position of third bright

Ex: Laser light (5000 Å) passes through a double slit arrangement 0.05 mm apart. The screen is 1 m away from slits.

a) Find fringe separation (=fringe width)
b) Find distance between 2nd bright and 3rd dark on opposite sides.

Ex: What can we do to obtain a better visible pattern in Young’s experiment?

{Explain effect of changing $\lambda$, d and L on $\Delta y$. Draw two example patterns for small and large d}

[Result: Slits closer, fringe centers distant. Slits distant, fringe centers closer]

Ex: Suppose while performing double-slit experiment, the space between the slits and the screen is filled with water. How does the interference pattern change?

Ex: A double-slit arrangement is illuminated first with red, then with blue light.

a) Which one has wider fringes?
b) Which one produces more fringes?

Ex: What happens if we use white light in place of monochromatic light in Young’s experiment?

Answer: Think about light of two color only (red-blue)

only red: only blue: together:

Intensity Distribution:

Rule: In double-slit interference, all fringes are equally bright and wide. [Actually we are neglecting diffraction effects for the time being. We take the slits sufficiently small themselves so as to make diffraction effects negligible. See N-slit diffraction]

6. Diffraction

Diffraction is bending of waves around an obstacle (barrier) [or spreading of waves passing through a narrow slit]

[We had seen diffraction with water waves]

Diffraction amount depends on $\frac{\lambda}{a}$ proportion.

If $a >> \lambda$, diffraction is negligible.
Same phenomenon is observable with light waves. Since $\lambda$ of light is very small, the opening must also be very small, something like 0.1 mm.

Single slit diffraction:

Most of the light energy is concentrated at the central maximum. Actually it is possible to say that all the light passing through the slit is spread as wide as the central maximum simply omitting the other bright fringes.

Actual pattern: We simply draw:

[We still have dark fringes although there is only one slit. Therefore light waves coming from different portions of the slit must be canceling]

If we divide the slit into two equal portions:

Now condition for first dark:

$$\delta = \frac{\lambda}{2} \Rightarrow \frac{a}{2} \sin \theta = \frac{\lambda}{2}$$

$$a \sin \theta = \lambda, \text{ (first dark)}$$

[First dark is important, because between two first darks we have the central bright, which receives nearly all the light energy passing through the slit]

if we divide the slit into 4, 6, 8, (even number) equal parts [and set $\delta=\lambda/2$ we will have $(a/4)\sin\theta=\lambda/2$, $(a/6)\sin\theta=\lambda/2, (a/8)\sin\theta=\lambda/2$] we get condition for other darks {explain relation between even number and dark}:

<table>
<thead>
<tr>
<th>Dark</th>
<th>Bright</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \sin \theta = m \lambda$</td>
<td>$a \sin \theta = (m+1/2) \lambda$</td>
</tr>
<tr>
<td>$\frac{a}{L} \frac{y_m}{m \lambda} = \frac{\lambda}{2}$</td>
<td>$\frac{a}{L} \frac{y_m}{m \lambda} = \frac{m}{2} \lambda$</td>
</tr>
<tr>
<td>$y_m = \frac{\lambda L}{a} m$</td>
<td>$y_m = \frac{\lambda L}{a} (m+1/2)$</td>
</tr>
<tr>
<td>$m=1,2,3, \ldots$</td>
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[We don’t have $m=0$ for central bright. Central bright is determined by position of first darks]

**Ex:** How have we found condition for brights? Sol: Divide the slit into 3, 5, 7, (odd number) parts: For first bright ($m=1$) we divide the slit into 3 equal portions. [Because “dividing” into 1portion gives us the central bright]

For first bright ($m=1$) $\delta = \frac{a}{3} \sin \theta = \frac{\lambda}{2}$ waves from two portions cancel but the remaining third portion illuminates the point on screen. So for first bright $m = 1 \Rightarrow a \sin \theta = \frac{\lambda}{2}$ waves

**Ex:** Derive fringe separation formula $\Delta y = ?$

$\Delta y = \frac{\lambda L}{a}$ [same between centers of brights and darks, only central bright $2\Delta y$]

**Ex:** 5000 A monochromatic light passes through a slit having 0.05 mm width. How much does it spread?
Sol: θ₁ for first dark:

\[ a \sin \theta = m \lambda \]

\[ m=1 \Rightarrow a \sin \theta = \lambda \]

\[ \sin \theta_1 = 5 \times 10^{-7} / 10^{-5} = 0.05 \]

\[ \theta_1 = 3° \]

If there wasn’t diffraction:

Ex: Monochromatic light (λ=6000 A) passes through a slit 0.1 mm wide and illuminates a screen 2 m away. Find width of central bright on screen.

Answer:

\[ 2\Delta y = 12 \text{ mm} = 1.2 \text{ cm} \]

If there wasn’t diffraction there would be a bright spot 0.1 mm wide on the screen.

Ex: Laser light having 6000 A wavelength passes through a slit 0.2 mm wide. On a screen placed 1 m away find

a) Distance from central line to second bright
b) Distance between second dark and third bright on different sides.

Ex: What are the effects of diffraction?

* We can not send a light ray along a straight path for a long distance. It will spread and lose intensity.

[Actually this is the case for any type of EM wave]

Ex: Diffraction from an edge (not a slit)

Ex: What is the minimum slit width for no diffraction minimum (dark fringe) to be observed?

Note: Boundary between geometric optics and wave optics:

There is no definite limit. Depends on:

- Width of light source
- Distance light travels

{Explain using the example below}

Ex: What is the maximum slit width for diffraction?

[Answer: If the light source is coherent, diffraction always occurs for all openings even if the slit is large. But according to formula: \( \sin \theta = \frac{\lambda}{a} \) (first dark), if \( a \gg \lambda \), then \( \theta \) is very small. So diffraction effect becomes negligible over small distances. The light follows nearly a straight path as wide as the slit for small distances if the slit dimension is large. But over large distances a small angle causes a large separation. If we are trying to send a 5 mm wide laser ray from earth to moon for example, the spreading of the beam will be \(~0.01°\), which is negligible at the beginning. But when it reaches the moon, the beam will be as wide as \(~80 \text{ km}\).]

7. Resolving Power

Two light sources are seen as a single source if they are far away enough. [Many bright dots in the night sky are actually star pairs – not single stars. Another example can be the two headlights of a car approaching from a distance]

The reason is diffraction. When light from the sources passes through the pupil of the eye, [which is a circular opening of \(~2-3 \text{ mm}\)] diffraction occurs. The retina acts as a screen.

If there was not diffraction:
Because of diffraction:

When $\theta$ gets smaller patterns overlap and seen as one:

Rule: Two sources seen as one when central bright of one pattern is on the first dark of the other.

Therefore:

First dark:

$$d \sin \theta = \lambda \Rightarrow \frac{\sin \theta}{a} = \frac{\lambda}{L} \quad \text{(just resolved)}$$

$$\frac{x}{L} = \frac{\lambda}{a} \quad \text{(seen as one source)}$$

[Actually these formulas are for slits, and can be used for a cat for example. For circular apertures we have a factor of 1.22 which we neglected here]

Ex: From what distance can we see two headlights of a car as two?
Distance between lights 1.5 m, take $\lambda = 5000 \text{Å}$. 
[The actual distance is much smaller due to other (such as atmospheric conditions. Diffraction is the ultimate limit in our ‘seeing’ power and since there are always other factors limiting our vision we are seldom limited by diffraction effects]

Ex: Explain why we can’t ever see an atom with normal light no matter how powerful a microscope we use. That is, explain how diffraction puts a limit to seeing small objects]

Ex: Explain why very large dishes are used for radiotelescopes.

8. Diffraction Grating
{Demo: N-slit diffraction java applet}

[The diffraction grating is a more useful device to analyse lights sources, because the interference maximums (bright fringes) are thin lines, making the measurements easier]

$$\delta = d \sin \theta$$

Therefore; $m$'th BRIGHT fringe:

$$d \sin \theta = m \lambda \quad (m = 0, 1, 2, 3 \ldots)$$

[Therefore we can use two slit formulas]

[We are not writing formula for dark fringe because dark fringes are actually wide dark bands between two bright lines]

Ex: A diffraction grating has 500 slits in 1 cm.
(a) Find slit spacing

Ex: Monochromatic light of 650 nm wavelength is incident on diffraction grating having $2\times 10^{-6}$ m slit spacing.
(a) How many bright lines will be observed?
(b) What is the angular position of the first diffraction fringe?

Ex: What is the path difference for the light waves forming the bright fringe at 30° from the central bright? Slit spacing of the diffraction grating is 0.05 mm.

Ex: A diffraction grating is illuminated by mixed red and blue light. Second bright of red coincides with the third bright of blue. Find $\lambda_{\text{blue}}$, if $\lambda_{\text{red}} = 6000 \text{ Å}$.

Ex: How many slits in 1 mm must a diffraction grating have, if it is to be used to analyse light having wavelength around 0.5 µm?
9. Interference in thin films

**Extreme case 1:**

In intermediate cases light is partly transmitted, partly reflected.

A soap bubble has two sides:

Light rays reflecting from two sides can cancel or reinforce according to phase difference between them.

**Rule:** Remember waves on a spring. Light waves have the same property.

**Extreme case 2:**

For destructive interference (dark):

\[2d \pm \frac{\lambda_{\text{film}}}{2} = (m-\frac{1}{2})\lambda_{\text{film}}\]

Looking from above:

[Looking from above means the light source and the observer are on the same side of the soap film]

\[2d = m\lambda_{\text{film}} \iff \text{Dark}\]

\[2d \left( m - \frac{1}{2} \right) \lambda_{\text{film}} \iff \text{Bright}\]

[Rule: light rays undergo 180° phase change upon reflection from an optically denser (with greater index of refraction) medium.]
For observer looking from below:

\[ 2d = \left( m - \frac{1}{2} \right) \lambda_{\text{film}} \quad \text{Dark} \quad m = 1, 2, 3, 4... \]

Therefore:

\[ 2d = m\lambda_{\text{film}} \quad \text{Bright} \quad m = 0, 1, 2, 3, 4... \]

Formulas changed place

Therefore:

One side dark ⇔ other side bright.

\textbf{Ex:} 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to be able to pass to other side.

\textbf{Ex:} 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to reflect back from the film surface.

\textbf{Ex:} 6000 A laser light is incident on a soap film (n=1.5). Find three different thicknesses the film might have, if the light is not reflecting back.

\textbf{Ex:} Film of changing thickness. [If you hold a soap film vertically lower side becomes thicker.]

Explain why we see many different colors over a soap film.

\textbf{Ex:} Lenses used in a camera are generally coated with a thin film of definite thickness. Why?